

Inconsistent Definitions of the Pressure-Coupled Response and the Admittance of Solid Propellants

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Nomenclature

a	speed of sound
A_b	admittance of the propellant surface
m	mass flow rate
M	Mach number
p	pressure
r	burn rate
R_p	pressure-coupled response
T	temperature
u	velocity
γ	ratio of specific heats
ρ	gas density
ρ_{solid}	density of the solid propellant

Superscripts

\wedge	unsteady quantity
$-$	mean quantity

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Introduction

When an acoustic wave is present in a solid propellant combustion environment, the mass flux from the combustion zone oscillates at the same frequency as the acoustics. The acoustic wave is either amplified or attenuated by the response of the combustion to the acoustic disturbance. When the acoustic wave is amplified, this process is called combustion instability. The amplification is quantitatively measured by a response function.

The ability to predict combustion stability for a solid propellant formulation is essential to the formulator to prevent or minimize the effects of instabilities, such as an oscillatory thrust. Unfortunately, the prediction of response values for a particular propellant remains a technical challenge.

Most predictions of the response of propellants are based on test data, but there are a number of questions about the reliability of the standard test method, the T-burner. Alternate methods have been developed to measure the response of a propellant, including the ultrasound burner, the magnetic flowmeter and the rotating valve burner, but there are still inconsistencies between the results obtained by these different methods.

Aside from the experimental differences, the values of the pressure-coupled responses obtained by different researchers are often compared erroneously, for the simple reason that inconsistencies in the definitions of the responses and admittances are not considered. The use of different definitions has led to substantial confusion since the first theoretical treatments of the problem by Hart and McClure in 1959.¹ The definitions and relations derived here seek to alleviate this problem.

Definitions

In general, a response function characterizes an unsteady property that results from another unsteady property. In the case of the pressure-coupled response, the response is a measurement of an unsteady property that is caused by an unsteady gas pressure above the surface of the propellant. Traditionally, the resultant unsteady property is the mass flux. However, it has also been defined as a combination of the mass flux and the gas temperature. There is some confusion in the literature about these definitions and the correct relations between the pressure-coupled response and the admittance of the propellant surface.

The most common definition of the response is the ratio of the non-dimensionalized mass flux to the non-dimensionalized pressure oscillation. This first definition is shown in Equation 1. A less common definition of the pressure-coupled response includes the gas temperature, as shown in Equation 2.²

$$Rp_1 = \frac{\hat{m}/\bar{m}}{\hat{p}/\bar{p}} \quad (1)$$

$$Rp_2 = \frac{\frac{\hat{m}}{\bar{m}} + \frac{\hat{T}}{\bar{T}}}{\frac{\hat{p}}{\bar{p}}} \quad (2)$$

The admittance is generally defined as a relation between the oscillatory velocity and the oscillatory pressure, as shown in Equation 3. This definition has been used extensively in the literature since its first use by Culick.³

$$A_{bl} = \bar{\rho}a \frac{\hat{u}}{\hat{p}} = \frac{\hat{u}/a}{\hat{p}/\gamma\bar{p}} \quad (3)$$

To arrive at the relation between the admittance and the definition of the response in Equation 1, it is first necessary to linearize the mass flux in the gaseous phase and the adiabatic relation, as shown in Equations 4 and 5, respectively. If these two relations and the definition of the admittance are then substituted into the first definition of the pressure-coupled response, the relation shown in Equation 6 is found.

$$\frac{\hat{m}}{\bar{m}} = \frac{\hat{u}}{\bar{u}} + \frac{\hat{p}}{\bar{p}} \quad (4)$$

$$\frac{\hat{p}}{\bar{p}} = \frac{1}{\gamma} \frac{\hat{p}}{\bar{p}} \quad (5)$$

$$Rp_1 = \frac{A_{b1} + M}{\gamma M} \quad (6)$$

The definition of the response that includes the temperature fluctuation has a slightly different relation to the admittance. The relation is obtained by linearizing mass continuity in both the solid and gaseous phases, and the ideal gas law. The mass flux of the solid phase in Equation 7 is equal to the density of the solid propellant multiplied by the burn rate. When this is linearized, the expression in Equation 8 is obtained.

$$\bar{m} = \rho_{\text{solid}} \bar{r} \quad (7)$$

$$\frac{\hat{m}}{\bar{m}} = \frac{\hat{r}}{\bar{r}} \quad (8)$$

The ideal gas law can also be linearized to obtain the expression in Equation 9. If the linearized mass fluxes in both the gaseous and solid phases (Equations 4 and 8) are combined and substituted into Equation 9, then the equation in Equation 10 is found.

$$\frac{\hat{p}}{\bar{p}} = \frac{\hat{p}}{\bar{p}} + \frac{\hat{T}}{\bar{T}} \quad (9)$$

$$\frac{\hat{m}}{\bar{m}} + \frac{\hat{T}}{\bar{T}} = \frac{\hat{u}}{\bar{u}} + \frac{\hat{p}}{\bar{p}} \quad (10)$$

Substituting Equation 10 back into Equation 2, a different relation between the pressure-coupled response and the admittance is obtained, as shown in Equation 11. Responses obtained from this relation are greater than the values obtained from Equation 6 by $1-1/\gamma$ for the same value of the admittance. This difference is on the order of 0.1 for most solid propellants, a small but significant difference.

$$Rp_2 = 1 + \frac{A_{b1}}{\gamma M} \quad (11)$$

A second definition of the admittance also leads to a yet another relation between the admittance and Equation 1. This definition of the admittance, shown here in Equation 12, has not been used with the definition of the pressure-coupled response. Culick defined the admittance as a function of the inverse of the acoustic impedance.⁴

$$A_{b2} = \frac{\hat{u}/\bar{u}}{\hat{p}/\bar{p}} \quad (12)$$

The relation between this definition of the admittance and Equation 1 is derived similarly to the Equation 6. The gaseous mass flux and the adiabatic relation are linearized and substituted into the definition of the response. Substituting in the different definition of the admittance gives the expression in Equation 13.

$$Rp_1 = A_{b2} + \frac{1}{\gamma} \quad (13)$$

This relation between the admittance and the response also produces significant differences in the value of the response for the same value of the admittance. In this case, the difference between the two responses is a factor of the admittance. Responses found

from Equation 6 could potentially be as much an order of magnitude greater than a response found using the relation in Equation 13 for the same value of the admittance. Fortunately, this relation is no longer commonly used to calculate the pressure-coupled response, although this is the definition of the admittance that is most commonly used in acoustic theory.

Conclusions

Although the two different definitions of the pressure-coupled response have lead to some confusion, the second definition of the response that includes the temperature flux oscillation is not commonly used today. The potential for confusion lies primarily in the use of different definitions of the admittance. Several of the experimental techniques use a measurement of the admittance to derive a response. It is vital that the admittance and pressure-coupled response be defined for the purpose of comparison to other experimental techniques.

References

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